

(e) sp hybridization and Hybrid orbitals

- Just LCAO
- LCAO : using s and p from same atom plus AOs from other atoms and do the matrix calculation

Some cases have strong mixings of s and p from the same atom Key idea

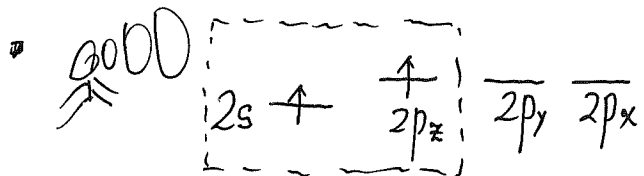
(24)

(this is a result of QM LCAO calculation)
and these mixed (hybrid) orbitals form bonds with AOs of other atoms

• Be H₂ exist (exp'tal fact). It is a linear molecule H-Be-H.

Be: 1s² 2s² all full!

But 2s — $\overline{2p_z}$ $\overline{2p_x}$ $\overline{2p_y}$ $\begin{matrix} \downarrow \\ \uparrow \end{matrix}$ not much different

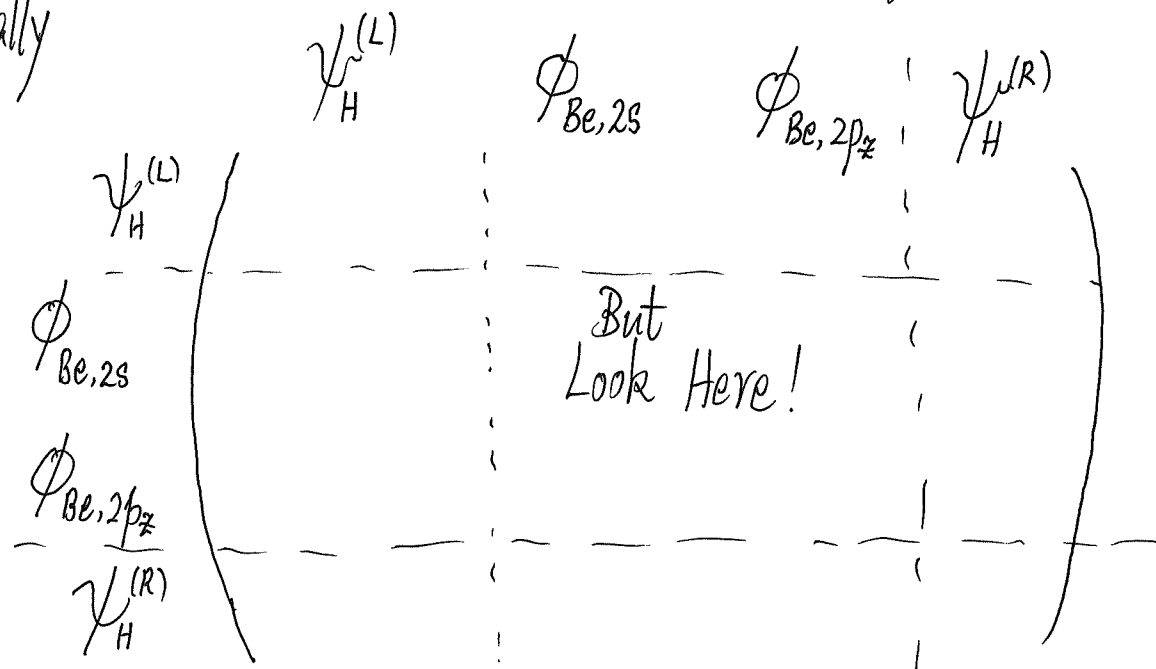


- Include Be's 2s and 2p_z AO's into LCAO consideration
- Hydrogen 1s from "left" AND 1s from "Right"

∴ 4x4

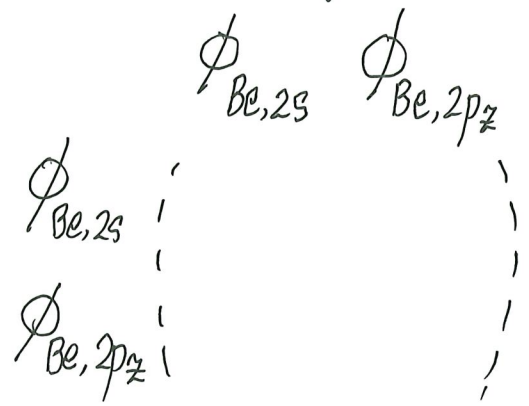
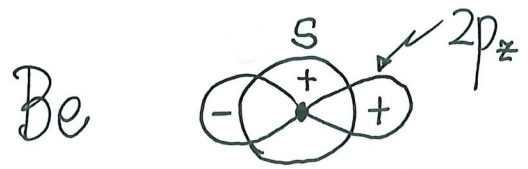
So, schematically

LCAO ⇒



Solve 4x4
⇒ Done!

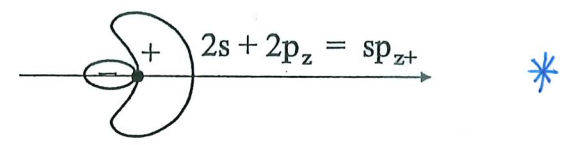
It is "legal" (mathematically) to manipulate [change basis] the 2x2 block formed by Be's AOs



Advantageous to form linear combinations: ["two in two out"]

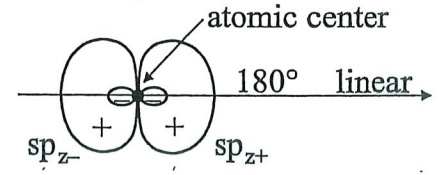
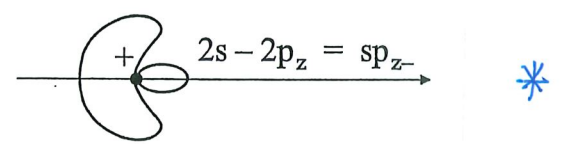
$$\psi_{\text{hybrid 1}}^{(\text{Be})} \sim \psi_{\text{Be},2s} + \psi_{\text{Be},2p_z}$$

same atom



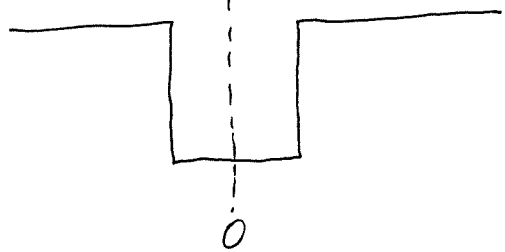
$$\psi_{\text{hybrid 2}}^{(\text{Be})} \sim \psi_{\text{Be},2s} - \psi_{\text{Be},2p_z}$$

sp hybridization

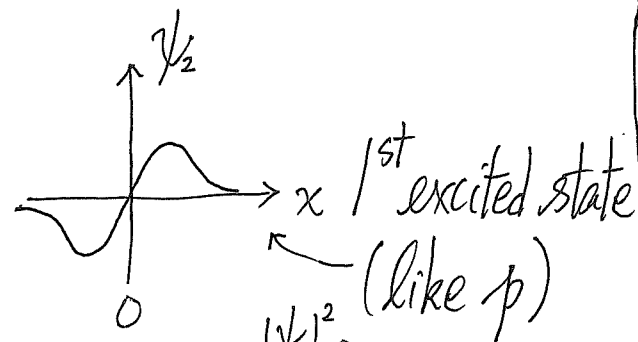
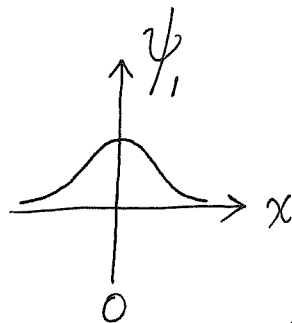


* Besides pointing at different directions, the two hybrid orbitals are equivalent. Highly directional, for stronger bonds with H-1s AOs on both sides

Analogy : 1D finite well



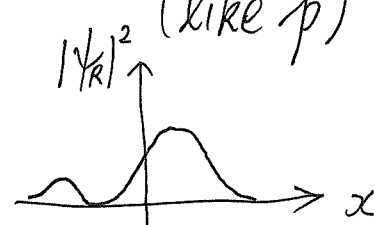
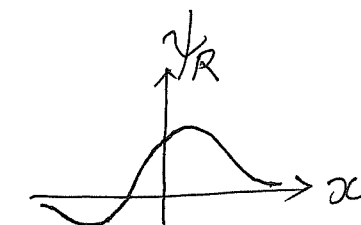
Ground state
(like s)



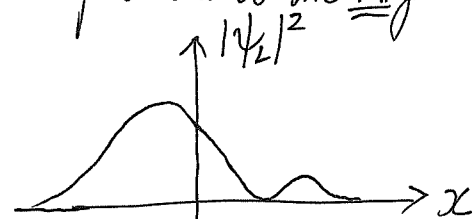
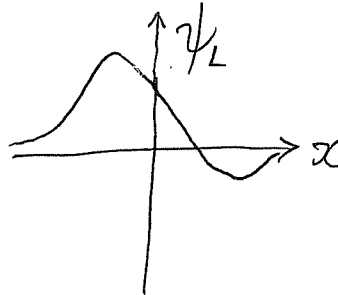
analogous to
hybrid orbitals

$\psi_R \propto \psi_1 + \psi_2$
[like $\psi_s + \psi_p$]

$\psi_L \propto \psi_1 - \psi_2$
[like $\psi_s - \psi_p$]



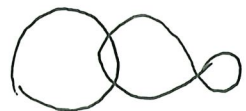
focused to the Right



focused to the Left

To prepare for forming bonds with atoms
in the right (using ψ_R) and in the left (using ψ_L)

What for?



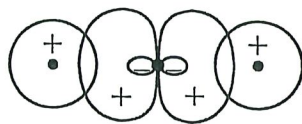
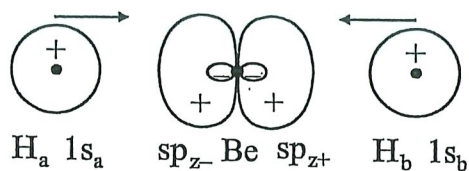
H (1s) Be [sp hybrid]

Be [sp hybrid]

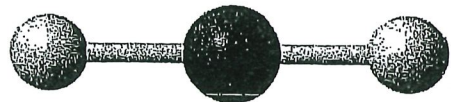


H (1s)

[Good overlap, strong bond]



BeH₂



[Energy is lowered]

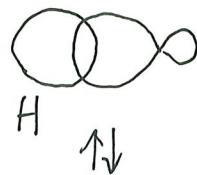
Mathematically speaking-

after changing basis to $\psi_{sp\#1}^{(Be)}$, $\psi_{sp\#2}^{(Be)}$
4x4 matrix becomes

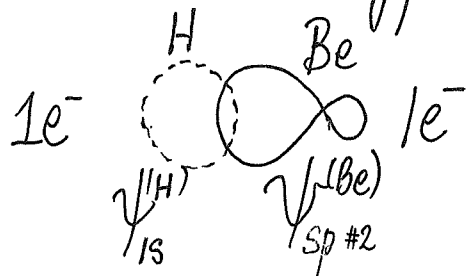
	$\psi_H^{(L)}$	$\psi_{sp\#2}^{(Be)}$	$\psi_{sp\#1}^{(Be)}$	$\psi_H^{(R)}$
$\psi_H^{(L)}$	$\epsilon_{1s}^{(H)}$	Δ	≈ 0	≈ 0
$\psi_{sp\#2}^{(Be)}$	Δ^*	$\epsilon_{hybrid}^{(Be)}$	≈ 0	≈ 0
$\psi_{sp\#1}^{(Be)}$	≈ 0	≈ 0	$\epsilon_{hybrid}^{(Be)}$	Δ^*
$\psi_H^{(R)}$	≈ 0	≈ 0	Δ	$\epsilon_{1s}^{(H)}$

2x2 block (upper)

2x2 block (lower)

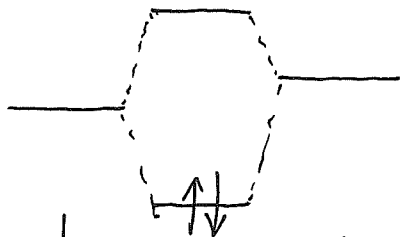


Gain Energy by forming bonds (BeH₂)

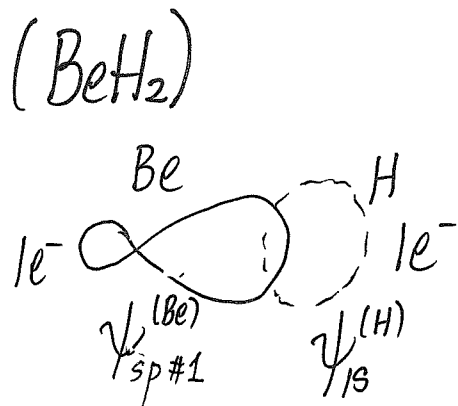
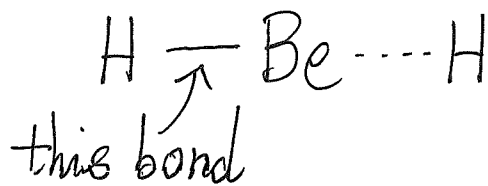


$$\begin{matrix} \psi_H^{(L)} \\ \psi_{sp\#2}^{(Be)} \end{matrix} \begin{pmatrix} \psi_H^{(L)} & \psi_{sp\#2}^{(Be)} \\ \epsilon_{1s}^{(H)} & \Delta \\ \Delta^* & \epsilon_{hybrid}^{(Be)} \end{pmatrix}$$

($|\Delta|$ becomes bigger)

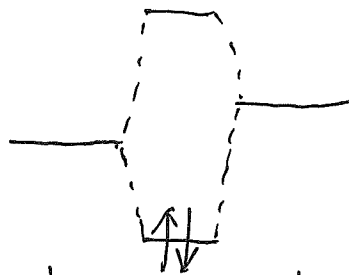


stronger pushing lowers energy

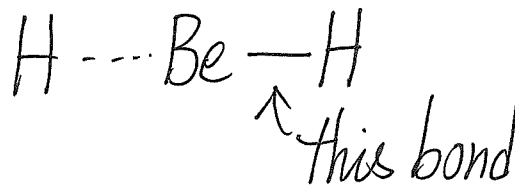


$$\begin{pmatrix} \epsilon_{hybrid}^{(Be)} & \Delta^* \\ \Delta & \epsilon_{1s}^{(H)} \end{pmatrix}$$

($|\Delta|$ becomes bigger)



stronger pushing lowers energy



Deeper: "Hybridization" is just a way to describe the wavefunction of Be, it is not an event

Let's see. $2s \text{---} \text{---} 2p \text{ (Be)} > \frac{2 \text{ electrons (in } 2s \text{ \& } 2p)}{\psi_s \quad \psi_p}$

↑ ↑
Be atomic orbitals

"One electron in ψ_s and one electron in ψ_p "

This is the physical situation (that we want to describe)

Description 1: Easy! 2 electrons in two states

$$\bar{\Psi}(1,2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_s(1) & \psi_p(1) \\ \psi_s(2) & \psi_p(2) \end{vmatrix} \quad (25)$$

satisfies anti-symmetric requirement

[in terms of atomic orbitals ψ_s and ψ_p (same atom)]

Description 2 : Introduce "Hybrid orbitals"

$$\psi_A = \frac{1}{\sqrt{2}}(\psi_s - \psi_p) \quad ; \quad \psi_B = \frac{1}{\sqrt{2}}(\psi_s + \psi_p) \quad (26)$$

[just two single-electron states mix & form two other single-electron states]

▪ ψ_A , ψ_B are the sp hybrid orbitals

With 2 electrons, we could as well say that "one electron in ψ_A and one electron in ψ_B "

$$\bar{\Phi}(1,2) = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_A(1) & \psi_B(1) \\ \psi_A(2) & \psi_B(2) \end{vmatrix} \quad (27)$$

[in terms of hybrid orbitals]

$\bar{\Psi}(1,2)$ vs $\bar{\Phi}(1,2)$? Are they different?

$$\begin{aligned}
 \bar{\Phi}(1,2) &= \frac{1}{\sqrt{2}} [\psi_A(1) \psi_B(2) - \psi_B(1) \psi_A(2)] \\
 &= \frac{1}{\sqrt{2}} \frac{1}{2} [(\psi_s(1) - \psi_p(1))(\psi_s(2) + \psi_p(2)) - (\psi_s(1) + \psi_p(1))(\psi_s(2) - \psi_p(2))] \quad (\text{using (24)}) \\
 &= \frac{1}{\sqrt{2}} \frac{1}{2} [\cancel{\psi_s(1)\psi_s(2)} + \psi_s(1)\psi_p(2) - \psi_p(1)\psi_s(2) - \cancel{\psi_p(1)\psi_p(2)} \\
 &\quad - \cancel{\psi_s(1)\psi_s(2)} + \psi_s(1)\psi_p(2) - \psi_p(1)\psi_s(2) + \cancel{\psi_s(1)\psi_p(2)}] \\
 &= \frac{1}{\sqrt{2}} [\psi_s(1)\psi_p(2) - \psi_p(1)\psi_s(2)] = \frac{1}{\sqrt{2}} \begin{vmatrix} \psi_s(1) & \psi_p(1) \\ \psi_s(2) & \psi_p(2) \end{vmatrix} \\
 &= \bar{\Psi}(1,2)
 \end{aligned}$$

They Describe the same wavefunction!
 (\therefore Hybridization is nothing fancy!)

• ψ_s, ψ_p are AOs of Be atom $\Rightarrow \int \psi_s^* \psi_s d\tau = 1$ and $\int \psi_s^* \psi_p d\tau = 0$
orthogonal

• $\psi_A = \frac{1}{\sqrt{2}} \psi_s + C_A \psi_p$; $\psi_B = \frac{1}{\sqrt{2}} \psi_s + C_B \psi_p$ (Find C_A, C_B systematically)

[Why $\frac{1}{\sqrt{2}}$ in front of ψ_s ? The s-orbital will NOT bias ψ_A or ψ_B]

Require ψ_A to be normalized $\Rightarrow \int \psi_A^* \psi_A d\tau = \frac{1}{2} \int \psi_s^* \psi_s d\tau + C_A^2 \int \psi_p^* \psi_p d\tau = 1$
 $\Rightarrow C_A = \frac{1}{\sqrt{2}}$

$$\therefore \boxed{\psi_A = \frac{1}{\sqrt{2}} (\psi_s + \psi_p)} \quad (28a)$$

Require ψ_B and ψ_A be orthogonal $\Rightarrow \int \psi_A^* \psi_B d\tau = \frac{1}{2} \int \psi_s^* \psi_s d\tau + \frac{1}{\sqrt{2}} C_B \int \psi_p^* \psi_p d\tau = 0$
(mathematically)

$$\therefore \boxed{\psi_B = \frac{1}{\sqrt{2}} (\psi_s - \psi_p)} \quad (28b)$$

$$\Rightarrow C_B = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$

$\psi_A \sim +\psi_p$ (point at $+\hat{z}$ direction); $\psi_B \sim -\psi_p$ ($-\hat{z}$ -direction) (180° apart)
 $\rightarrow \vec{V}_A$ $\vec{V}_A \cdot \vec{V}_B = -1$ $\vec{V}_B \leftarrow$

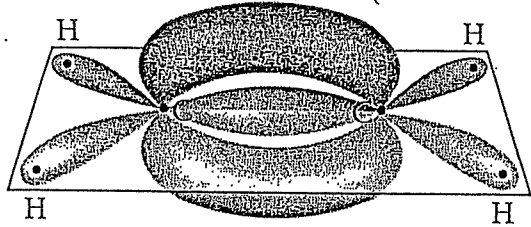
in physical space

The deeper description and approach of getting the hybrid orbitals (sp in this case) are applicable to sp^2 and sp^3 hybridization

- s won't bias any hybrid orbital
- Hybrid orbitals are themselves normalized and mutually orthogonal
- spread out in space for bonding with other atoms.

(f) C₂H₄ (ethylene) and sp² hybridization

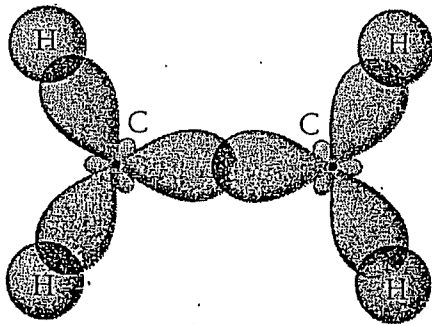
double (one σ and one π) bond



(a)

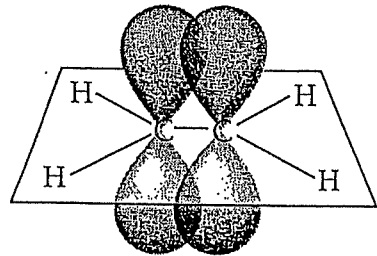
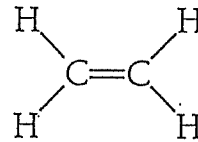
(a) The ethylene (C₂H₄) molecule. All the atoms lie in a plane perpendicular to the plane of the paper. (b) Top view, showing the sp² hybrid orbitals that form σ bonds between the C atoms and between each C atom and two H atoms. (c) Side view, showing the pure p_x orbitals that form a π bond between the C atoms.

Top View



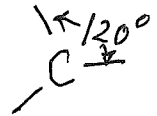
(b)

Ethylene



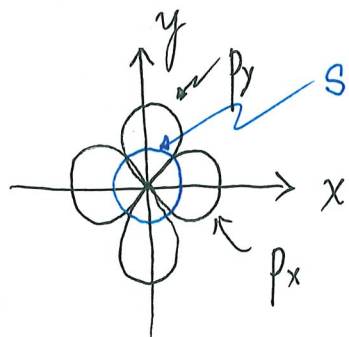
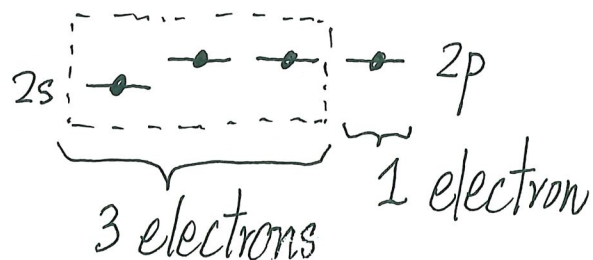
(c)

Again,
highly
directional



[120° between bonds.]

The two electrons in the two pure p-orbitals form a π-bond

Carbon: $[1s]^2 2s^2 2p^2$ 

ψ_{2s} , ψ_{2p_x} , ψ_{2p_y} to form 3 sp^2 hybrid orbitals

Let ζ_1 , ζ_2 , ζ_3 be the hybrid orbitals

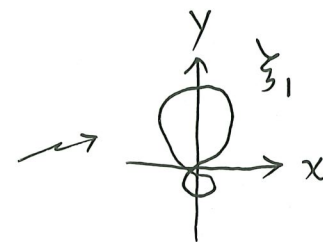
$$\zeta_1 = \frac{1}{\sqrt{3}} \psi_{2s} + b_1 \psi_{2p_y} + 0 \cdot \psi_{2p_x} \quad [\text{OK to put } \zeta_1 \text{ in } +\hat{y} \text{ direction, still general}]$$

$$\zeta_2 = \frac{1}{\sqrt{3}} \psi_{2s} + b_2 \psi_{2p_y} + c_2 \psi_{2p_x}$$

$$\zeta_3 = \frac{1}{\sqrt{3}} \psi_{2s} + b_3 \psi_{2p_y} + c_3 \psi_{2p_x}$$

$$\zeta_1 \text{ normalized} \Rightarrow b_1 = \sqrt{\frac{2}{3}}$$

$$\therefore \zeta_1 = \sqrt{\frac{1}{3}} \psi_{2s} + \sqrt{\frac{2}{3}} \psi_{2p_y}$$



$$\zeta_2 \text{ orthogonal to } \zeta_1 \Rightarrow b_2 = -\sqrt{\frac{1}{6}}$$

$$\zeta_2 \text{ itself normalized} \Rightarrow c_2 = \sqrt{\frac{1}{2}}$$

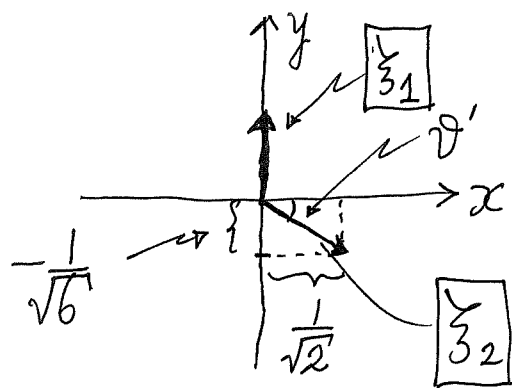
$$\therefore \zeta_2 = \sqrt{\frac{1}{3}} \psi_{2s} - \sqrt{\frac{1}{6}} \psi_{2p_y} + \sqrt{\frac{1}{2}} \psi_{2p_x} \quad (29)$$

Which direction does ξ_2 point at?

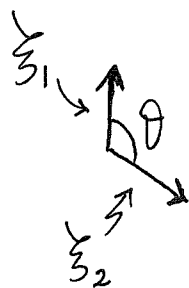
Lazy way (works)

$$\xi_1 = \underbrace{\frac{1}{\sqrt{3}} \psi_{2s}}_{\text{no direction}} + \underbrace{\sqrt{\frac{2}{3}} \psi_{2p_y}}_{\text{like a vector in } \hat{y}\text{-direction}}$$

$$\xi_2 = \underbrace{\frac{1}{\sqrt{3}} \psi_{2s}}_{\text{no direction}} - \frac{1}{\sqrt{6}} \psi_{2p_y} + \frac{1}{\sqrt{2}} \psi_{2p_x} \sim -\frac{1}{\sqrt{6}} \hat{y} + \frac{1}{\sqrt{2}} \hat{x}$$



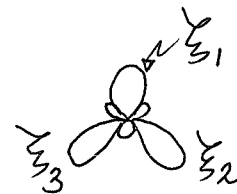
$$\tan \theta' = \frac{\frac{1}{\sqrt{6}}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{3}} \Rightarrow \theta' = 30^\circ$$



$$\theta = 90^\circ + 30^\circ = 120^\circ$$

[Again, QM is behind directionality]

Exercise: Find ξ_3 and the direction it points at.

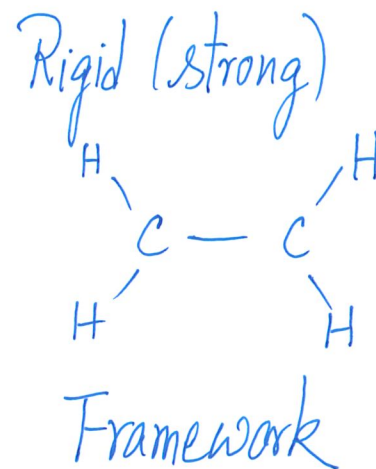
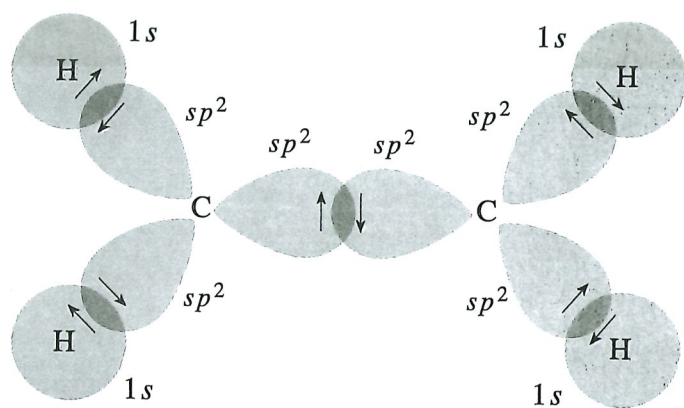
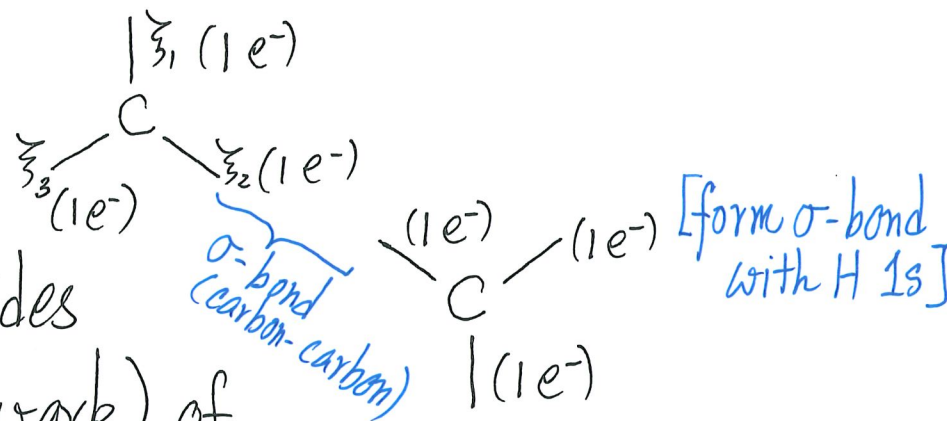


Consequences

• Plane of hybrid orbitals

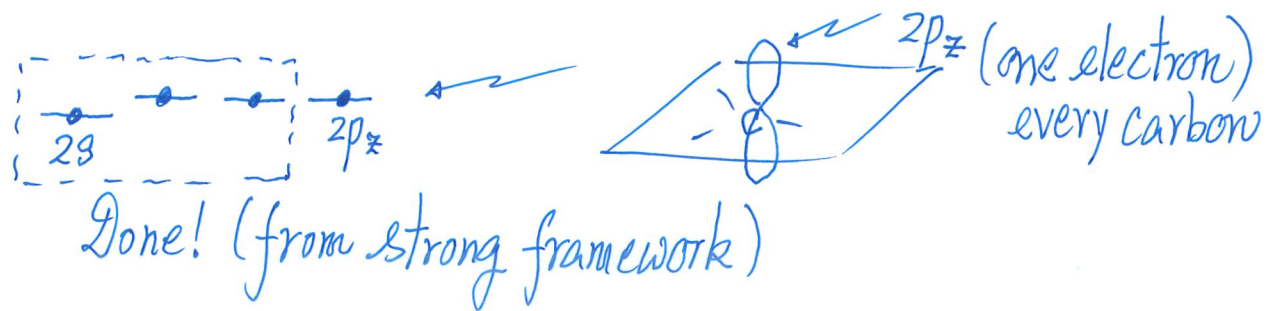
sp^2 hybridization provides
the skeleton (framework) of

forming strong σ -bonds (direction) and dictates the structure

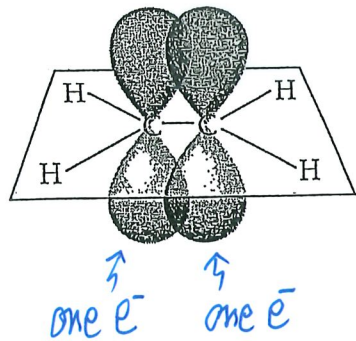


The (planar) σ -bond framework of an ethene molecule.

What's left?

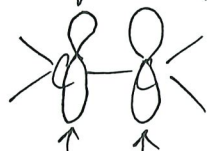


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Quick QM thinking on π -bond

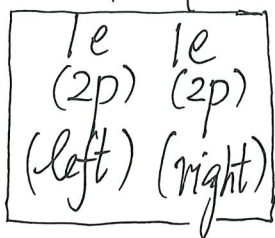
After forming the strong σ -bonds skeleton



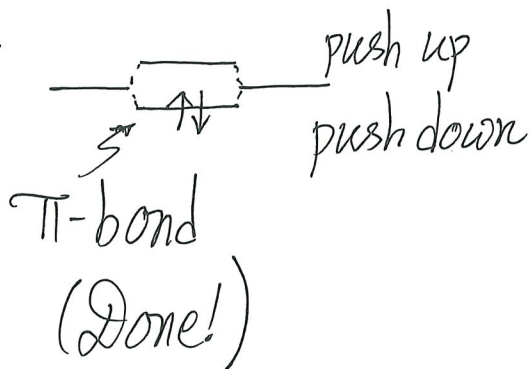
$$\begin{pmatrix} \epsilon_{2p} & \tilde{\Delta} \\ \tilde{\Delta}^* & \epsilon_{2p} \end{pmatrix}$$

(LCAO, 2x2 matrix problem)

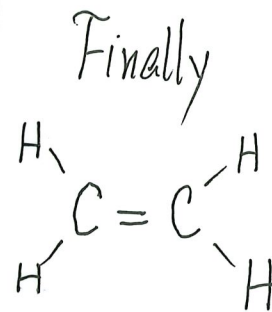
These electrons are called π electrons



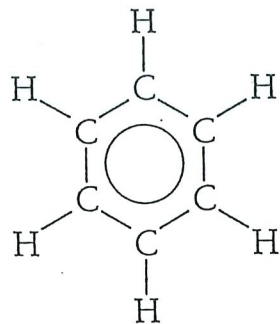
$$\tilde{\Delta} = \int \phi_L^* \hat{H} \phi_R d\tau$$



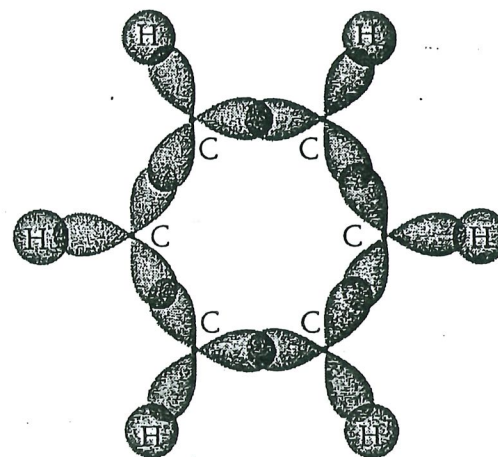
Notes: $\tilde{\Delta} = \int \phi_L^* \hat{H} \phi_R d\tau$ can't be big (small region having non-zero integrand)
 \Rightarrow pushing is smaller \Rightarrow weaker than σ -bond.




(g) Benzene C_6H_6 : What is the "O" inside  ?

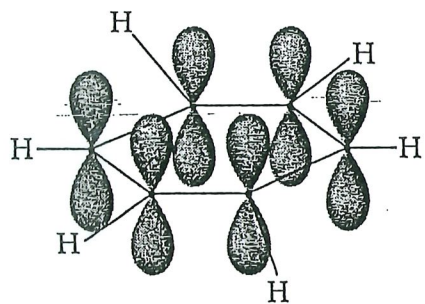


these strong bonds locked in the electrons



- sp^2
- σ bonds form framework 

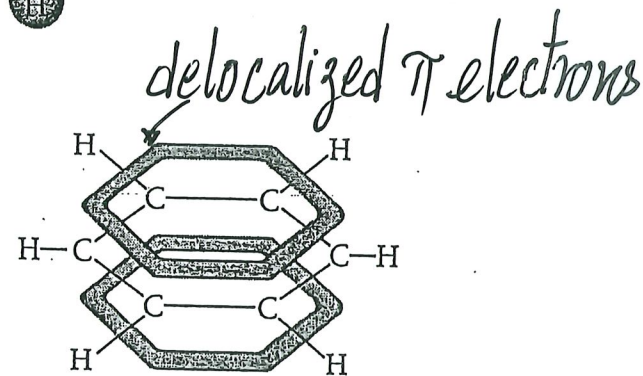
▪ Each C has a $2p_z$ (perpendicular to plane) with one electron

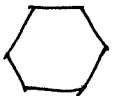


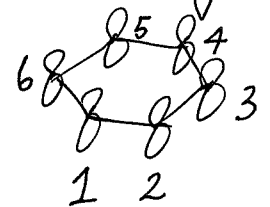
Behavior of six π -electrons?



[Quantum Mechanics Approach?]



▪ Framework  is rigid and strong

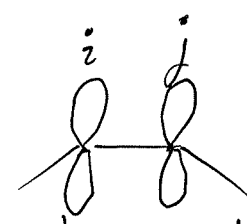
▪ Q: Behavior of p_z electrons  (one electron each, six p_z electrons)

▪ How do they form bonds? LCAO of p_z AO's (c.f. C_2H_4 case)

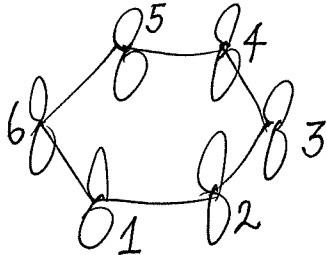
Think like a physicist! $\Psi = c_1 \phi_{p_{z1}} + c_2 \phi_{p_{z2}} + c_3 \phi_{p_{z3}} + c_4 \phi_{p_{z4}} + c_5 \phi_{p_{z5}} + c_6 \phi_{p_{z6}}$

↳ Some \hat{H} (don't even need to write it down explicitly)

▪ Formally, $|H_{ij} - ES_{ij}| = 0$ ($i, j = 1, 2, \dots, 6$) gives 6 values of E for 6 p_z electrons to fill in

▪ Practically, $S_{ii} = 1$
 $S_{ij} \approx 0$  and H_{ij} more important
"Not much overlap"

- H_{ii} is mainly governed by the atomic ϵ_{2p} energy
 $\Rightarrow H_{11} = H_{22} = \dots = H_{66} = \epsilon$ (diagonal elements)

-  "1"s interacts with "2" and "6" Only
 nearest neighbors of "1"
 "3", "4", "5" are farther away \Rightarrow ignore H_{13}, H_{14}, H_{15}

- For two nearest neighboring 

$$H_{ij} = \underbrace{V}_{\text{same between two nearest neighbors}} \quad [\text{nearest-neighbor interaction}]$$

- Need only two terms:
 ϵ and V
 (or α) (or β) [$\beta < 0$ typically]
- in standard notations

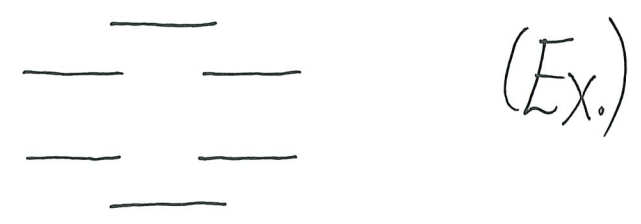
$$\begin{array}{c}
 \textcircled{1} \\
 \textcircled{2} \\
 \textcircled{3} \\
 \textcircled{4} \\
 \textcircled{5} \\
 \textcircled{6}
 \end{array}
 \begin{array}{c}
 \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \quad \textcircled{4} \quad \textcircled{5} \quad \textcircled{6} \\
 \left| \begin{array}{cccccc}
 \varepsilon - E & V & 0 & 0 & 0 & V \\
 V & \varepsilon - E & V & 0 & 0 & 0 \\
 0 & V & \varepsilon - E & V & 0 & 0 \\
 0 & 0 & V & \varepsilon - E & V & 0 \\
 0 & 0 & 0 & V & \varepsilon - E & V \\
 V & 0 & 0 & 0 & V & \varepsilon - E
 \end{array} \right| = 0
 \end{array}$$

gives six values of E

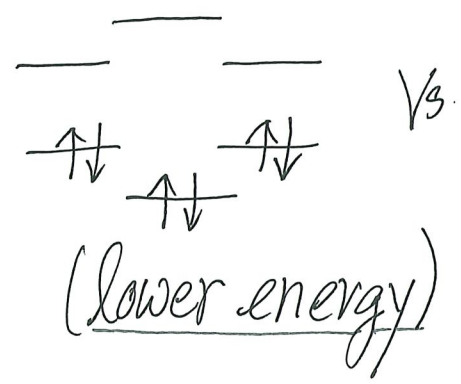
or written as a 6x6 matrix problem

$$\begin{pmatrix}
 \varepsilon & V & 0 & 0 & 0 & V \\
 V & \varepsilon & V & 0 & 0 & 0 \\
 0 & V & \varepsilon & V & 0 & 0 \\
 0 & 0 & V & \varepsilon & V & 0 \\
 0 & 0 & 0 & V & \varepsilon & V \\
 V & 0 & 0 & 0 & V & \varepsilon
 \end{pmatrix}
 \begin{pmatrix}
 C_1 \\
 C_2 \\
 C_3 \\
 C_4 \\
 C_5 \\
 C_6
 \end{pmatrix}
 = E
 \begin{pmatrix}
 C_1 \\
 C_2 \\
 C_3 \\
 C_4 \\
 C_5 \\
 C_6
 \end{pmatrix}
 \quad (30)$$

- Six values of E are lined up as
(there are MO's for π electrons)



- There are 6 π electrons
(Pauli Principle)

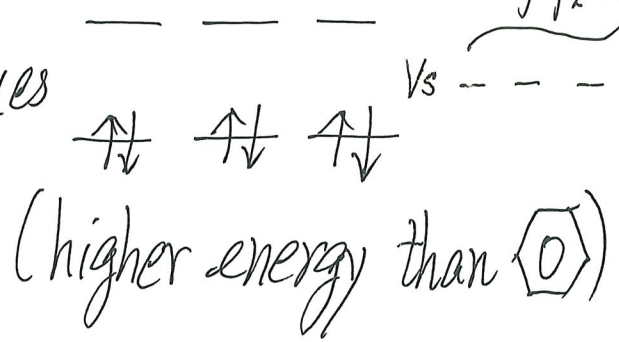


vs.
if p_z 's don't couple

- How about ?

$$\begin{pmatrix} \epsilon & V & 0 & 0 & 0 & 0 \\ V & \epsilon & 0 & 0 & 0 & 0 \\ 0 & 0 & \epsilon & V & 0 & 0 \\ 0 & 0 & V & \epsilon & 0 & 0 \\ 0 & 0 & 0 & 0 & \epsilon & V \\ 0 & 0 & 0 & 0 & V & \epsilon \end{pmatrix}$$


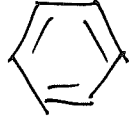
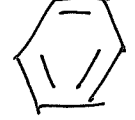
\Rightarrow Eigenvalues



vs.
if p_z 's don't couple

"1" only interacts with "2", etc.



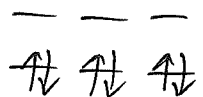
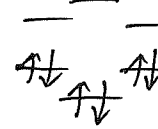
[becomes three identical, 2x2 problems]

•  has a lower energy than  and 

"1" to "2" & "6", then to "3" and "5", and to "4"

⇒ Delocalizing the electrons can lower the energy ↪ Benzene
↪ Metals

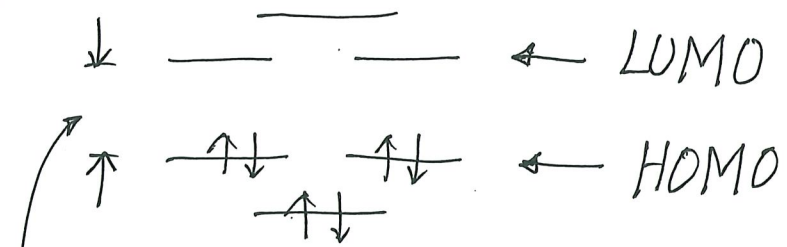
Delocalizing Energy = Energy lowered by delocalizing the electrons

= Energy difference between  and 
 

• This is the Hückel Theory (1931)

Only H_{ii} and nearest-neighbor H_{ij}
(ϵ) (V)
(α) (β)

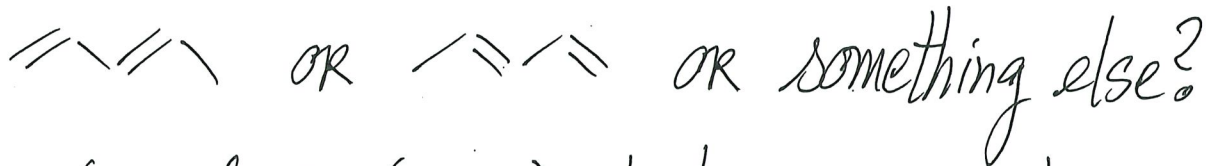
LUMO - HOMO Difference



LUMO-HOMO energy difference[†]

controls material's absorption/
emission frequency (optical properties)

How about butadiene?



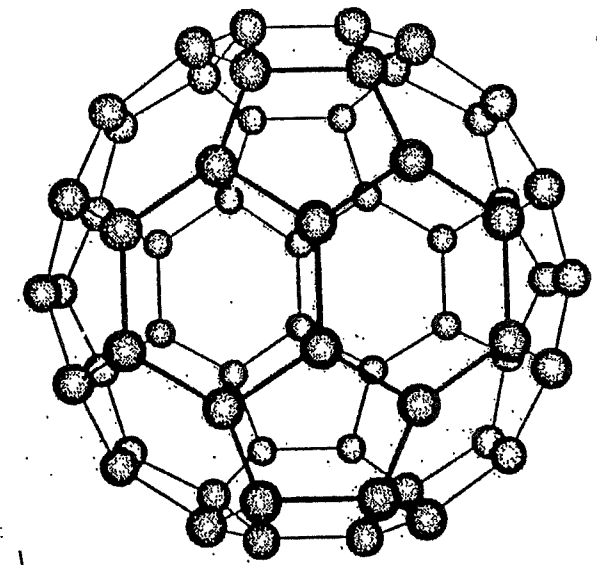
Behavior of p_z (OR π) electrons in graphene, C_{60} ?

Delocalizing is key to physics of metals

[†] The LUMO-HOMO difference becomes the band gap in insulators/semiconductors

(h) sp² hybridization Plus π electrons in Graphene and C₆₀

C₆₀ molecule ["Bucky Ball"] [Kroto and Smalley: 1996 Nobel Chemistry Prize]



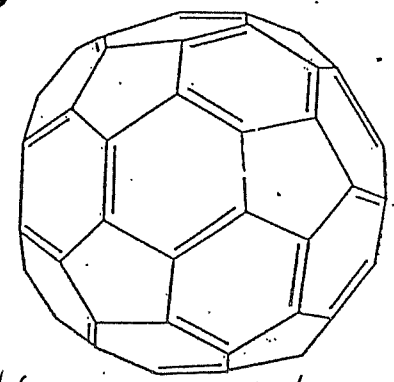
20 hexagons
12 pentagons

sp² hybridization
 π electrons

diameter $\approx 7 \text{ \AA}$

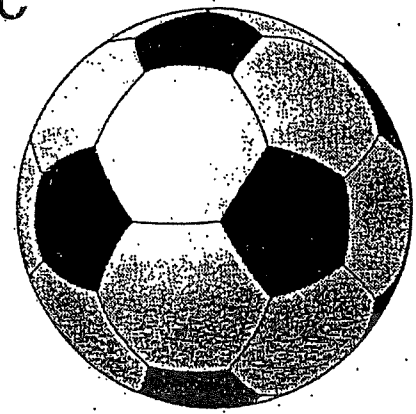
Formally, a
Truncated
Icosahedron
↑
(二十面體)
(twenty plane faces)

b Schematic



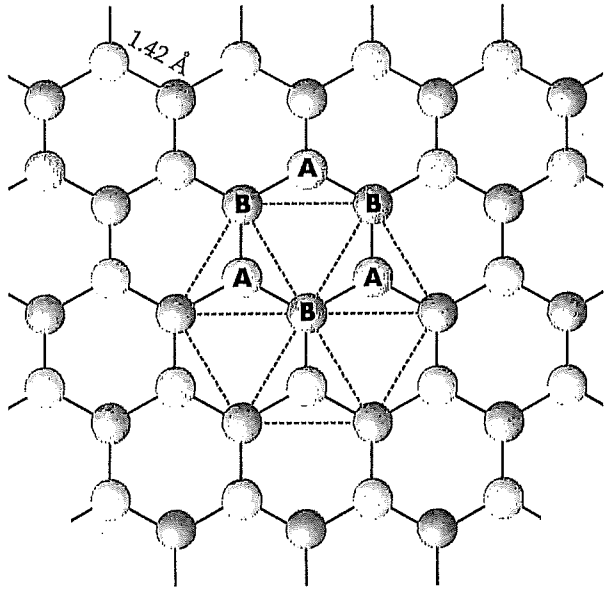
delocalized π -electrons

c

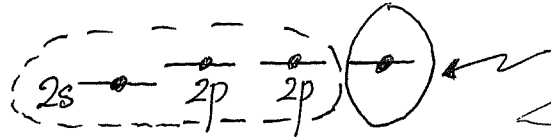


Graphene: A single atomic layer of carbon

[A. Geim and K. S. Novoselov: 2010 Nobel Physics Prize]



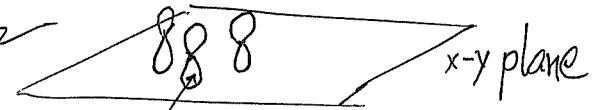
- All are carbon atoms
- Labels A and B for atoms that see different environments



these 3 electrons formed covalent bonds with neighboring atoms

locked into bonds

they can't move around after forming bonds



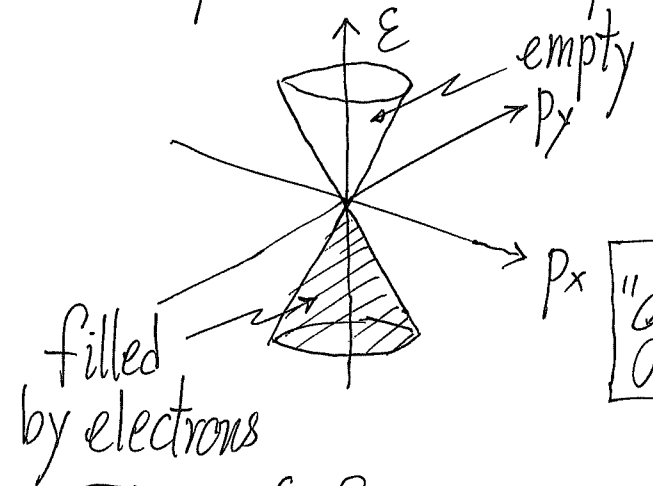
each carbon has one electron in a $2p_z$ state \updownarrow perpendicular to the graphene sheet (π electrons)

π electrons can move on sheet!

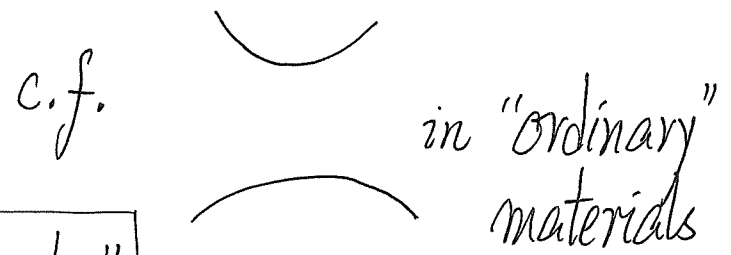
The π electrons in Graphene behave unusually!

- They move around "above and below" the sheet
- In a solid, electron states form bands described by $\overset{\text{energy}}{\downarrow} \underset{\text{momentum of electron}}{\uparrow} \epsilon(p)$

▪ Graphene has $\epsilon(p)$ of the form[†]



"gapless semiconductor"



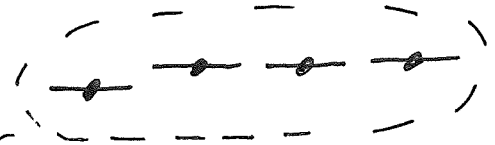
Form of $\epsilon = vp$ [c.f. $\epsilon^2 = m^2c^4 + c^2p^2 \Rightarrow \epsilon = cp$ for massive particle]

[like massless fermions!] (Table-top QED system?)

[†] This is LCAO applied to solids, often called tight-binding model in energy band theory (solid state physics)

(i) sp^3 Hybridization [Pauling 1928, Nobel Chemistry Prize 1954]

▪ CH_4 , diamond, silicon, ... (semiconductor industry)

▪ Carbon (again)  $2s^2 2p^2 \rightarrow \underbrace{2s^1 2p_x^1 2p_y^1 2p_z^1}_{\text{all in}}$

→ 4 electrons available for forming bonds

→ one s AO and three p AOs combine to form

Four Hybrid Orbitals (sp^3 hybrid orbitals)

Formally, construct Hybrid Orbitals

$$\psi_i = \frac{1}{\sqrt{4}} \psi_s + b_i \psi_{p_x} + c_i \psi_{p_y} + d_i \psi_{p_z} \quad (30)$$

($i=1, 2, 3, 4$) require ψ_i to be normalized and mutually orthogonal

The end results are

$$\psi_1 = \frac{1}{2} (\psi_s + \psi_{p_x} + \psi_{p_y} + \psi_{p_z})$$

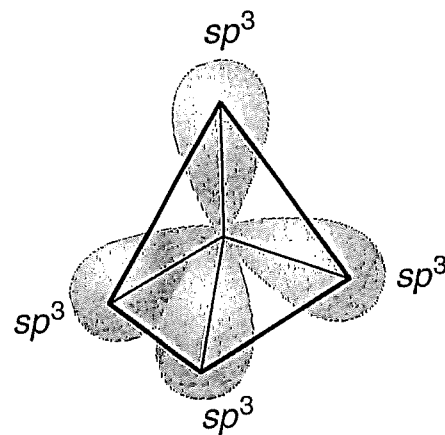
$$\psi_2 = \frac{1}{2} (\psi_s - \psi_{p_x} - \psi_{p_y} + \psi_{p_z})$$

$$\psi_3 = \frac{1}{2} (\psi_s + \psi_{p_x} - \psi_{p_y} - \psi_{p_z})$$

$$\psi_4 = \frac{1}{2} (\psi_s - \psi_{p_x} + \psi_{p_y} - \psi_{p_z})$$

sp^3
hybrid
orbitals

(31)



Angle between orbitals
= 109.5° (Ex.)

These are the four sp^3 Hybrid Orbitals

▪ Highly directional

▪ Like four vectors in

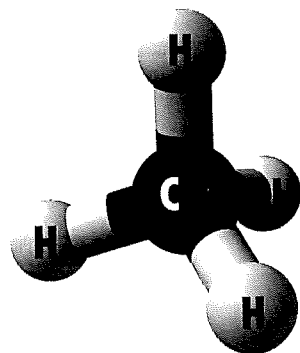
$$\left\{ \begin{array}{l} (+1, +1, +1) \quad [\hat{x} + \hat{y} + \hat{z}] \\ (-1, -1, +1) \quad [-\hat{x} - \hat{y} + \hat{z}] \\ (+1, -1, -1) \quad [\hat{x} - \hat{y} - \hat{z}] \\ (-1, +1, -1) \quad [-\hat{x} + \hat{y} - \hat{z}] \end{array} \right\}$$

directions (or rotated as a whole)

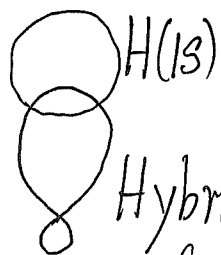
What for?

To lower energy by forming bonds

Methane CH₄

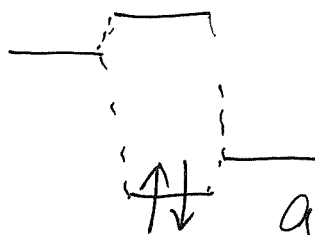


For each hybrid orbital (say ψ_i), there is one electron ready for bonding.



Hybrid orbital from carbon

$$\begin{matrix} \langle \psi_i | & | \psi_i \rangle & | \psi_{H,1s} \rangle \\ \langle \psi_{H,1s} | & \begin{pmatrix} \epsilon_i & \Delta \\ \Delta^* & \epsilon_{H,1s} \end{pmatrix} \end{matrix}$$



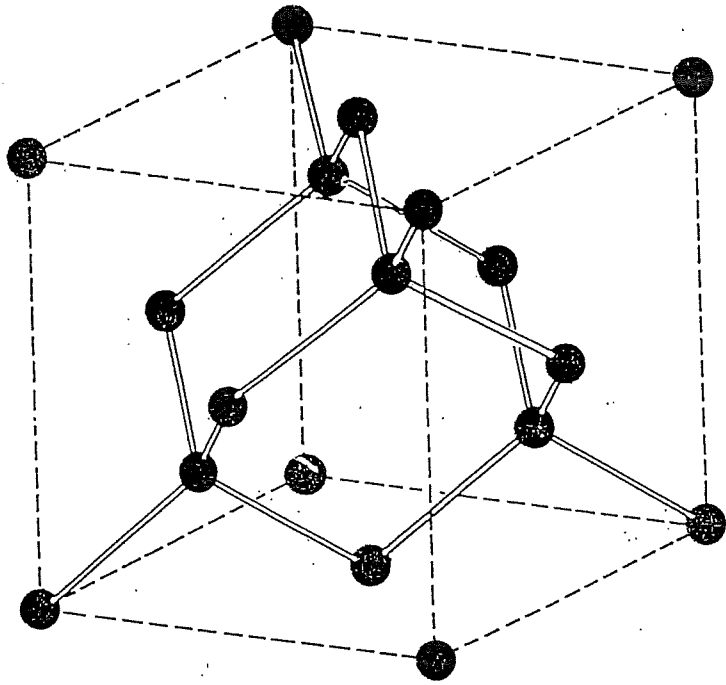
gain energy for each σ -bond formed

[Same physics works!]

Δ is made bigger by

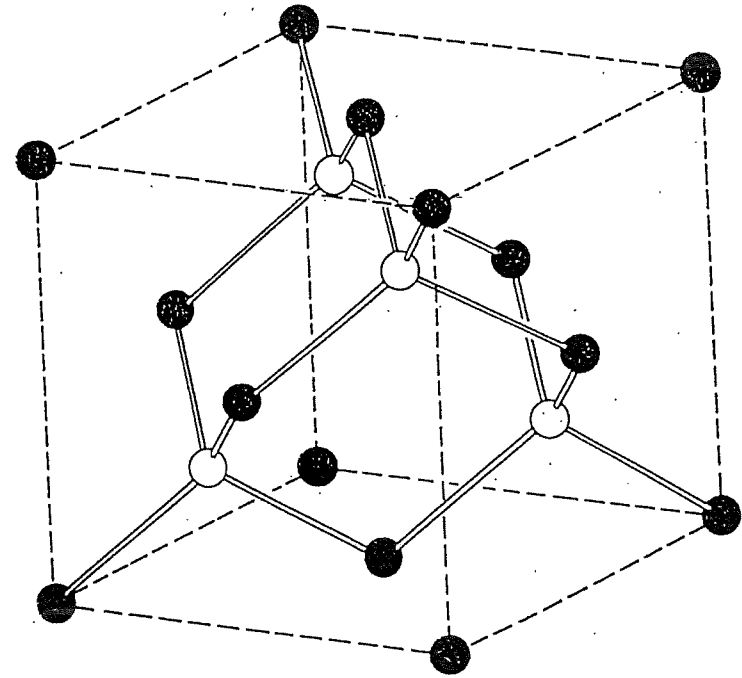
- Hybrid orbital focuses electron towards $\psi_{H,1s}$
- Hybrid orbitals spread electrons farther apart

sp^3 Hybridization governs many semiconductors' structure



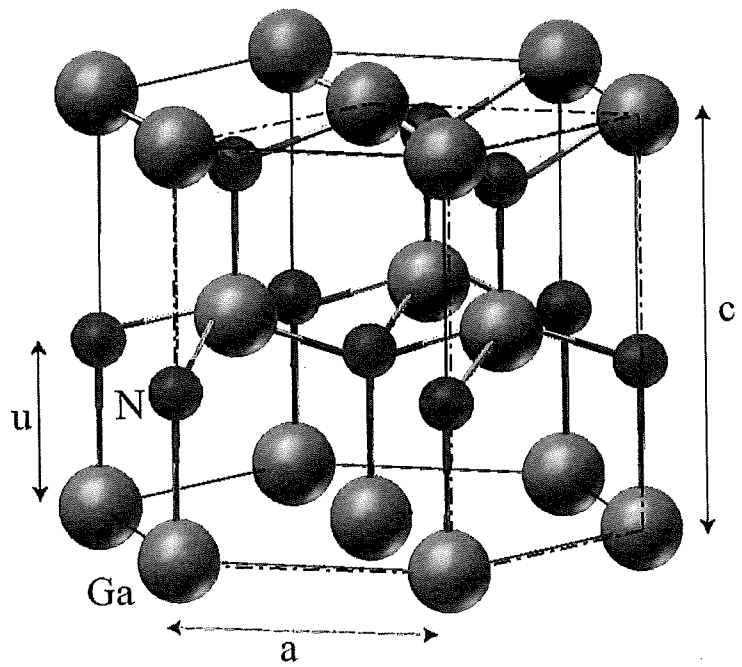
Diamond Structure

- Same atom at every \bullet
[e.g. carbon, silicon, germanium]



Zinc Blende Structure

- \bullet A atom
 - \circ B atom
- Most important materials! \rightarrow [e.g. GaAs (III-V semiconductor)
CdTe (II-VI Semiconductor)]



Wurtzite Structure

E.g. GaN, ZnO
 blue LED & blue laser
 [2014 Nobel Physics Prize]
 used in your mobile phone

sp^3 Hybridization gives crystal structures of important semiconductors
AND the structures determine the bands and band gaps

A bit of Quantum Mechanics takes us into the fascinating world of molecules, solids, and materials science.

Take-Home Messages

- QM, LCAO-MO, "Variational Method \leftrightarrow Matrix Math" (often 2×2) explain much of the world of materials (molecules & solids)
- Put binding (ionic, covalent bonds), hybridization, σ and π bonds, Huckel theory (π -electrons) under one roof (just QM)
- A little idea (solid foundation needed though) that does a lot of things!

Robert S. Mulliken and Friedrich Hund[†] developed Molecular Orbital Theory

- 1927 at Göttingen (worked in Born's group)
- Mulliken developed LCAO

1966 Nobel Chemistry Prize to Mulliken

"for his fundamental work concerning bonds and the electronic structure of molecules by the molecular orbital theory."

[Hund was left out, but Mulliken said he would have gladly shared the Prize with Hund.]



Mulliken (left) and Hund

[†] Hund's contributions also included the Hund's rules.

An interesting comment by Dirac in 1929 on physics and chemistry

P. A. M. Dirac, *Proceedings of the Royal Society A* 123, 714 (1929)
 714 (p. 714-733)

Quantum Mechanics of Many-Electron Systems.

By P. A. M. DIRAC, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received March 12, 1929.)

§ 1. *Introduction.*

The general theory of quantum mechanics is now almost complete, the imperfections that still remain being in connection with the exact fitting in of the theory with relativity ideas. These give rise to difficulties only when high-speed particles are involved, and are therefore of no importance in the consideration of atomic and molecular structure and ordinary chemical reactions, in which it is, indeed, usually sufficiently accurate if one neglects relativity variation of mass with velocity and assumes only Coulomb forces between the various electrons and atomic nuclei. The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems without too much computation.

Already before the arrival of quantum mechanics there existed a theory of atomic structure, based on Bohr's ideas of quantised orbits, which was fairly successful in a wide field. To get agreement with experiment it was found

Should appreciate that up to now, work and progress continue to be made in how to perform QM calculations more accurately and efficiently for the understanding of molecules (many biology related)